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THE INTERACTION BETWEEN A SINGLE PARTICLE AND AN OSCILLATING FLUID

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Abstract—The motion of single particles in a sinusoidally oscillating flow field as the simplest case of a turbulent motion of an eddy is investigated theoretically and experimentally. Since complete knowledge about the interaction between the particles and the surrounding fluid is lacking, three models are evaluated for the interpretation of the sphere's behaviour. The different phenomena of the particle behaviour in an oscillating flow field are described, and the validity of the model which agrees most closely with the experimental results is limited.

INTRODUCTION

In flowing suspensions solid particles are suspended due to the turbulent fluctuation velocity of the eddies in the fluid. Theoretical considerations on topics related to this transport process are given by e.g. Tchen (1947), Soo (1956), Liu (1956), Friedländer (1957), Hinze (1959), Chao (1964), Ahmadi & Goldschmidt (1970). Starting with the equation of motion for a spherical solid particle with diameter d and density ρ_p moving with an eddy, a balance of the inertial, buoyancy, gravitational and drag forces yields the equation for the instantaneous velocity $\mathbf{v}$ of the particle in an eddy moving with the instantaneous velocity $\mathbf{u}$.

With the assumption of very low relative velocities between the sphere and the fluid i.e. low Reynolds numbers, Re, defined using the kinematic viscosity ν,

$$Re = \frac{|\mathbf{u} - \mathbf{v}|}{\nu} d, \qquad [1]$$

the convective acceleration terms can be disregarded in the Navier Stokes equations. Basset (1888, 1910), Boussinesq (1885) and Oseen (1927) have solved these differential equations which apply to a sphere translatorily moving in a fluid at rest with high acceleration but low velocity. The expression derived by them for the resistance force due to the interaction between the flow field and the sphere consists of three terms:

(i) the viscous resistance for steady flow patterns described by Stokes, with a coefficient $c_D = 24/Re$,

(ii) the added mass known from the theory of inviscid flows for unsteady conditions, with a coefficient $c_a = 0.5$,

(iii) the integral term, with a coefficient $c_h = 6$, generally known as Basset term in literature.

Resolving the drag force into these terms, the balance of forces yields:

$$\frac{\pi}{6} d^3 \rho_p \frac{d\mathbf{v}}{dt} - \frac{\pi}{6} d^3 \rho_f \frac{d\mathbf{u}}{dt} + \frac{\pi}{6} d^3 (\rho_p - \rho_f)\mathbf{g} + c_D \frac{\pi}{8} d^2 \rho_f |\mathbf{v} - \mathbf{u}| \cdot (\mathbf{v} - \mathbf{u}) + c_a \frac{\pi}{6} d^3 \rho_f \frac{d}{dt}(\mathbf{v} - \mathbf{u}) + c_h \frac{\pi}{4} d^2 \sqrt{\frac{\nu}{\pi}} \int_{t_0}^{t} \frac{(d/dt^*)(\mathbf{v} - \mathbf{u})}{\sqrt{t - t^*}} dt^* = 0 \qquad [2]$$

where $\mathbf{g}$ is the gravitational acceleration, ρ_f the fluid density, and t the time. This is, however, only academic due to incomplete knowledge about the unsteady drag for higher relative velocities, i.e. the coefficients c_D, c_a and c_h are unknown. Torobin & Gauvin (1959, 1960, 1961) reviewing the relevant research state, conclude that there exists no expression for the acceleration influence on the drag coefficient generally valid for all cases of motion.

Odar & Hamilton (1964) analyzed the force acting on an oscillating sphere in a fluid at rest. For higher relative velocities, they tried to embed the steady state drag coefficient as a function of the Reynolds number into the unsteady drag coefficient, and determine the coefficients of the added mass and the Basset term from their experimental data. The structure of the Basset term and the structure of the added mass are the same as in creeping flow, but the coefficients of these terms are now functions of an instantaneous acceleration number

$$Ac = \frac{(\mathbf{u}-\mathbf{v})^2}{\left| d \cdot \frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{u}-\mathbf{v}) \right|}, \qquad [3]$$

but not of the Reynolds number.Their result is

$$c_a = 1.05 - \frac{0.066}{Ac^2 + 0.12}$$
$$c_h = 2.88 + \frac{3.12}{(Ac+1)^3}. \qquad [4]$$

These coefficients asymptotically approximate the values of the creeping flow, i.e. $c_a = 0.5$ and $c_h = 6$, for very small acceleration number. With higher acceleration number the significance of the Basset term on the sphere's motion decreases, whereas the influence of the added mass increases.

The drag of a guided sphere, as studied experimentally by Odar (1964), in the general case behaves differently from a sphere which is freely movable since the degrees of freedom of a guided sphere are restricted. Moreover, the empirical coefficients given by Odar are correlated with experimental data which refer to Reynolds numbers less than 62.

Because of difficulties determining the unsteady drag coefficient experimentally, theoretical numerical solutions of the Navier–Stokes equations with higher relative velocities for unsteady conditions have been attempted (e.g. LeClair & Hamiliec 1970), but have not yet given satisfying results; at least these numerical attempts have not led to a quantitatively evaluable relation.

TEST OF THE EQUATION OF MOTION

To test [2], the motion of a single sphere in a vertical oscillating flow field having the sinusoidal velocity

$$u = A\omega \cdot \cos \omega t, \qquad [5]$$

with amplitude A and angular frequency ω has been investigated. The particle's behaviour is predicted with each of the following modifications of [2].

(a) In the quasisteady equation of motion, the unsteady drag is assumed to be the steady drag with a constant added mass. Thus, quasisteady state means consequently, that the drag coefficient c_D in [2] conforms with that for steady state at the same actual Reynolds number. Moreover a constant coefficient $c_a = 0.5$ of added mass and no Basset term, i.e. $c_h = 0$, result from these assumptions.

(b) In a second modification, the Basset term in creeping flow with a constant coefficient $c_h = 6$ has been added to the terms of the quasisteady equation of motion.

(c) The third model includes the quasisteady drag coefficient c_D for higher Reynolds number, added mass and Basset term coefficients which are functions of the acceleration number according to [4] given by Odar.

Since in a vertically oscillating flow field all forces acting on the sphere are assumed to be vertical, [2] gives a prediction only for the vertical component of the sphere's motion. Thus the model equations are non-linear ordinary differential equations yielding the vertical velocity of the sphere. It should be noted that there are approximate analytic solutions of the quasisteady state equation given by several workers, e.g. Houghton (1966, 1968a,b), Molerus (1964), Baird *et al.* (1967), Schöneborn (1973). At this point the predictions of the different models should be verified.

EXPERIMENTAL WORK

In the literature there are some publications (Wagenschein 1921; Brush *et al.* 1963; Ho 1964; Baird *et al.* 1967; Tunstall & Houghton 1968; Field 1968; El-Tawil 1969; Kuychoukov, Molinier & Angelino 1970; Molinier, Kuychoukov & Angelino 1971) treating the behaviour of spherical particles in a sinusoidally oscillating flow field. It is found that the quasisteady model describes the time average fall velocity well for low frequencies with a rather great amplitude. According to Ho's (1964) experiments, the fall velocity of individual spheres in an approximately sinusoidal fluid field at frequencies of 3.5 to 6.8 Hz and amplitude of 1.27 to 5.08 cm show a preponderantly good conformity among experimental values.

This can be expected for the quasisteady model. The ratio of the average fall velocity, u_0, in an oscillating flow field to the fall velocity w_f in a fluid at rest is called the retarding effect. In figure 1 the retarding effect has been plotted vs the amplitude A. The curves represent solutions of the quasisteady equation of motion for a change in the direction of relative velocity and for an unchanged direction. The horizontal straight line with hatchings gives the transition from one to the other type of motion. The points are experimental data given by Ho. The Reynolds number Re_{ruh} is based on the settling velocity w_f in a fluid at rest.

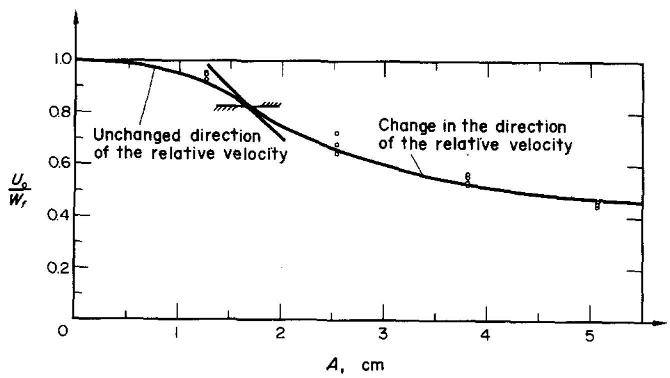

Figure 1. A comparison of the quasisteady equation of motion and experimental data of Ho ($d = 1$ mm, $f = 5.2$ c/s, $\rho_p/\rho_f = 7.72$, $Re_{ruh} = 390$).

With respect to solids dispersion, it should be noted that the turbulent fluctuations of the eddies include components with higher frequencies. Thus, the angular frequency of the turbulence spectrum can reach values up to approximately 50 Hz. However, the quasisteady equation of motion underlying the approximate solution holds for low frequencies of the flow field only; with increasing frequency the time for the wake development decreases. Therefore, the variation of drag is dependent on the wake development and was investigated experimentally.

Figure 2 shows a schematic diagram of the equipment. The cylinder filled with a fluid is moved sinusoidally by means of a cam. A single sphere is released electromagnetically. With dark-field illumination, the motion of the sphere reflected from a rotating mirror (Hertel, Affeld & Clauss, 1969) is photographed. This rotating mirror, having the speed f_m, is needed to interject a virtual transverse motion to the trajectory of the sphere.

The normal **n** to the mirror and the axis of rotation enclose an angle of 3 degrees. A rotation of the reflection plane results in a picture which is distorted vertically and laterally. The mapping of

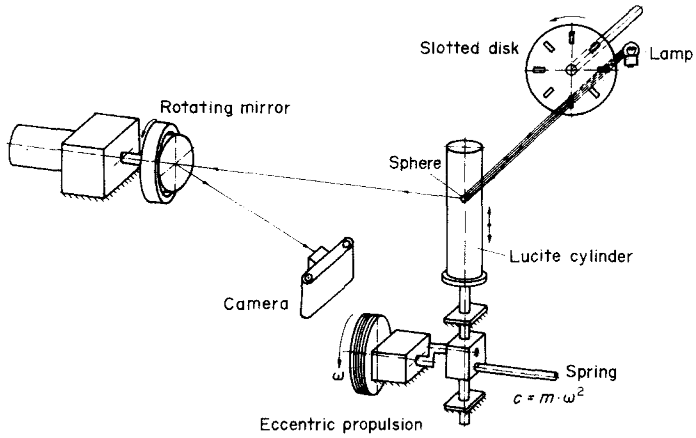

Figure 2. The experimental set-up.

a bright point, fixed on the axis of the cylinder, is cyclical. The motion of a point, moving with constant velocity, yields a cycloid. A rotating slotted disk interrupts the ray 240 times per second, providing a time scale for the sphere's motion. For each test, one photograph is shot containing the entire information about the sphere's motion.

In figure 3, x_2 and x_3 are the coordinates of the real plane of the sphere's motion, y_2 and y_3 are the coordinates of the image plane of the rotating mirror, which is relevant to the photograph. Points with an equal time difference taken from a photograph are drawn in figure 3. The oscillating portion of the sphere's motion causes the deviation of these points from a cycloid. Since the virtual distortion of the trajectories, caused by the rotating mirror, is known, it is possible to calculate the position of the sphere at a definite time.

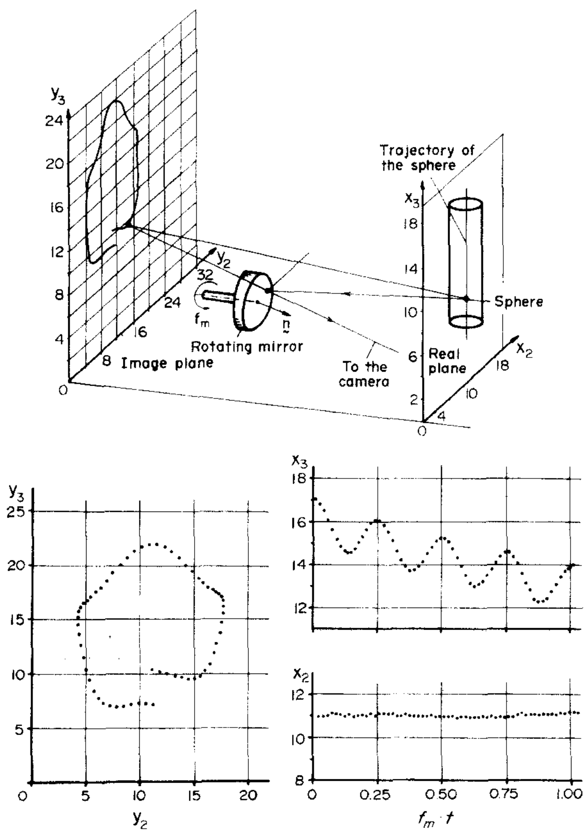

Figure 3. The evaluation of the sphere's position.

The regenerated coordinates (x_2, x_3) of the sphere's trajectory as a function of the time t referred to the time of one mirror revolution $1/f_m$ is also drawn in figure 3. The lateral coordinate x_2 shows that the sphere's trajectory has been along a straight line. A Fourier analysis of the vertical coordinate x_3 of the trajectory as a function of the time yields the time average fall velocity, and the oscillating part of the sphere's velocity.

The sphere's trajectory in an oscillating flow field is, in the general case, different from the trajectory in a fluid at rest. The flow past a sphere at steady inflow described, e.g. by Taneda (1956) is symmetrical to the direction of inflow below a Reynolds number of about 300. In this case the wake consists of one permanent vortex ring which begins to oscillate when a Reynolds number of about 130 is reached. For a Reynolds number of more than 300, the wake becomes labile and subsequent vortex rings separate from the sphere. This periodic vortex shedding leads to an unsteady periodic flow past the sphere as e.g. observed by Magarvey & Bishop (1961), Magarvey & Blackford (1962), Magarvey & MacLathy (1965) and Achenbach (1974). As a consequence of the shedding, asymmetries with respect to the upstream–downstream axis of the sphere appear in the wake. The wake reacting on the sphere causes a fluctuation in the motion of a freely movable sphere. As the configuration of the wake is asymmetric, there are deflecting forces acting on the sphere, which cause a lateral motion of a freely movable sphere. The formation of a vortex is connected with a energy loss in the translational motion, consequently the actual velocity of a freely movable sphere is reduced.

In figure 4 the left photograph shows a sphere's trajectory in a fluid at rest. The vortex shedding produces an insignificant lateral deviation of the trajectory from a straight line. The other photographs in figure 4 show trajectories of a sphere in an oscillating flow field. In an oscillating flow field the vortex shedding frequency can be synchronized with the frequency of the flow field. The sphere performs alternating, regular lateral fluctuations around a vertical line. The photographs in the middle of figure 4 show this form of trajectory. Viets (1971) has observed that during the acceleration portion of the sphere's fall in a fluid at rest, vortex shedding is connected with the lateral motion of a freely movable sphere. A certain time is needed for the development of the wake and vortex shedding. With increasing frequency, time does not suffice for a formation of a vortex system past the sphere. Finally, the vortex shedding is suppressed and the sphere moves with strictly rectilinear motion along a vertical line, because of the symmetrical wakes before and behind the sphere. The photograph on the right hand side in figure 4 shows such a trajectory.

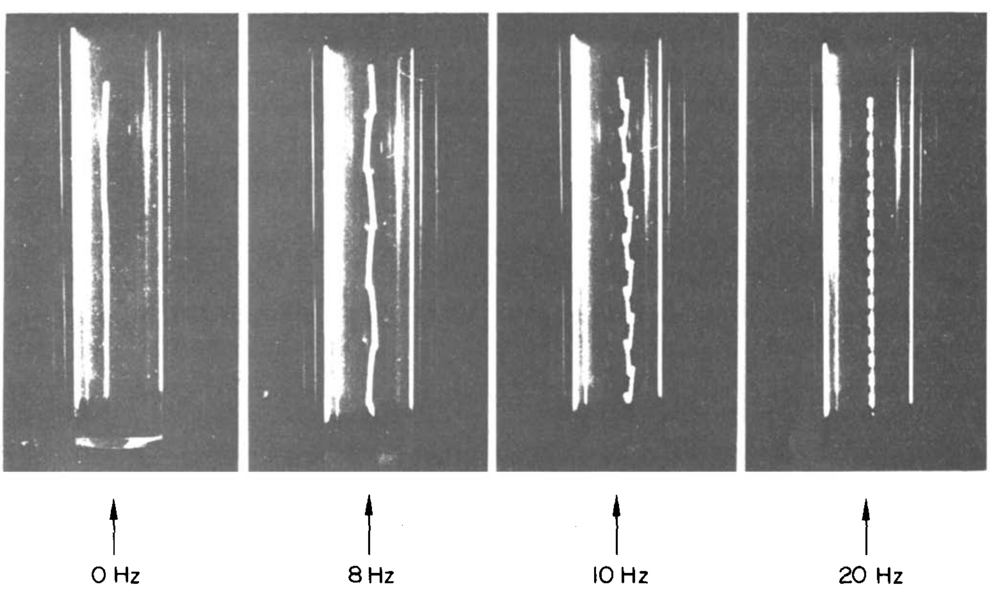

Figure 4. The sphere's trajectory in an oscillating flow field.

In the frequency regime near the natural vortex shedding frequency, the peculiar behaviour of a sphere distinctly affects the values of the average fall velocity quantitatively. In figure 5 experimental data for the retarding effect as a function of the frequency are shown for three spheres consisting of the same material but having different diameters.

The experimental values of the retarding effect decrease conspicuously for relative low values of the frequency. The natural vortex shedding takes place at a frequency of close to 8 Hz as can be estimated from experimental results given by Goldburg & Florsheim (1966) and Achenbach (1974). Each shedding vortex leads to a drag fluctuation. In the frequency regime near the shedding frequency, a behaviour similar to resonance occurs, since even in a fluid at rest the system formed with the sphere and the wake of the sphere tends to self-excited oscillations. In the frequency regime in which self excited oscillations are possible, the forced oscillation of the sphere requires an additional input of energy, which leads to an increased drag and a significantly decreased fall velocity. A regular lateral sphere's motion marks this regime optically. For a cylinder kept between springs laterally and fixed in the direction of inflow the phenomenon of flow induced oscillation in connection with an increasing drag near the vortex shedding frequency has been observed by Sedrak (1971). With increasing frequency the trajectory is stabilized because of the suppressed vortex shedding. The experimental data approach the theoretical values for an equation of motion including the unsteady drag given by Odar transformed to the relative motion. In figure 5, curve c denotes values according to this model. The theoretical values according to a quasisteady equation of motion are too low. The failure of a quasisteady equation represented by curve a in figure 5 is reasonable, since a quasisteady model implies a free vortex shedding, which is not correct in this case.

Since the separation of vorticies gives a non-linear contribution to the drag (von Karman & Rubach 1912), a quasisteady equation of motion yields time average settling velocities too low in

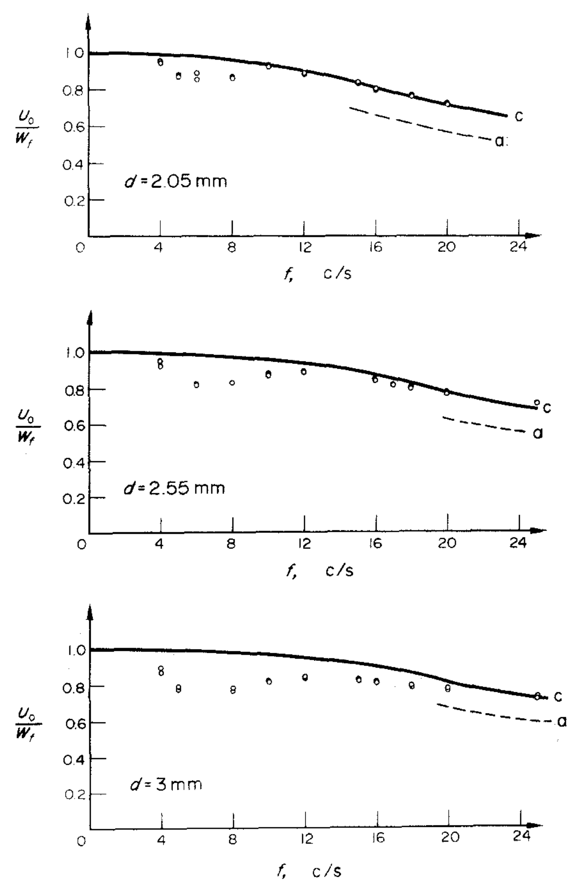

Figure 5. The retarding effect vs the frequency (A = 8 mm, ν = 1 cSt, ρ_p/ρ_f = 1.43).

the regime of suppressed vortex shedding. The approach to the calculated curve is markedly sharper with decreasing diameter of the sphere.

Figure 6 shows the retarding effect as a function of the frequency for spheres without vortex shedding in the wake. The decrease in the experimental values of the retarding effect with reference to the theoretical values for the separation of vortices disappears in this case.

The experimental data can be described essentially with an equation of motion represented by curve c, which includes the drag according to Odar. A quasisteady equation of motion, curve a, is not right at all. Additionally, an equation of motion which includes besides the quasisteady drag, the full Basset term of the creeping flow does not lead to good results (curve b).

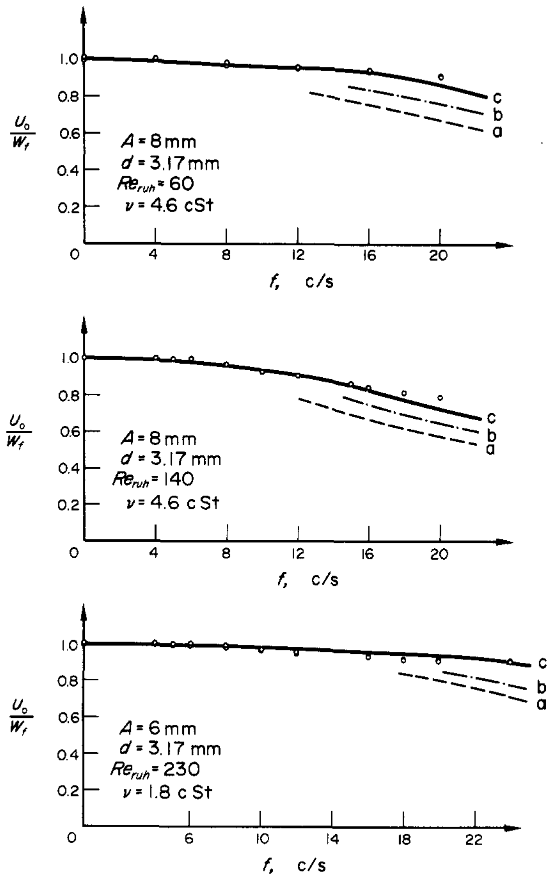

Figure 6. The retarding effect vs the frequency for spheres without vortex shedding from the sphere.

The amplitude ratio i.e. the oscillatory part of the sphere's motion to the velocity amplitude of the flow field is shown in figure 7. A comparison between the experimental and the theoretical predicted values admits the same conclusion as before, whereupon from these equations the equation with the unsteady drag given by Odar predicts the sphere's behaviour with the smallest error on condition that there is no vortex shedding in the wake.

The experiments show that vortex shedding has a decisive effect on the sphere's motion in an oscillating flow field. If the Reynolds number Re_{ruh} takes sufficiently large values that a regular vortex sheet exists, there are three different regimes of the sphere's behaviour. These regimes differ in the ratio of the frequency for the flow field to the vortex shedding frequency.

The periodical unsteadiness in the wake caused by the vortex shedding is described with the Strouhal number $f_s d/w_f$ (f_s represents the natural vortex shedding frequency), whereas the Reynolds number characterizes the properties of the steady flow. Under steady flow condition for a fixed sphere the Strouhal number is a function of the Reynolds number only. To specify the

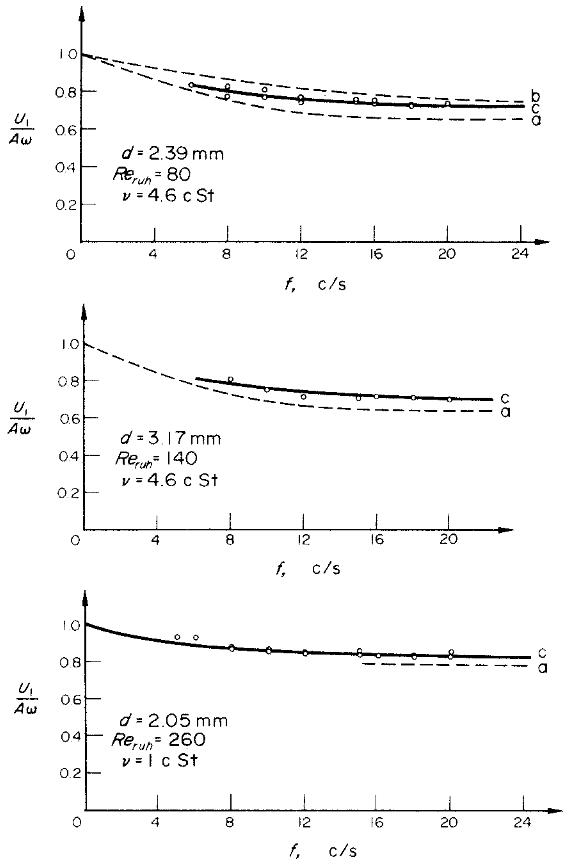

Figure 7. The amplitude ratio versus the frequency ($A = 8$ mm).

sphere's behaviour in an oscillating flow field, a modified Strouhal number, fd/w_f should be used. If the frequency of the flow field f is much less than the natural vortex shedding frequency f_s, i.e. the modified Strouhal number fd/w_f is small, then an intense influence of the unsteady flow field on the sphere's wake cannot be expected. The experimental results of Baird *et al.* (1967), Ho (1964), Molinier (1971) cover the frequency regime including only the beginning of the resonance.

This existing experimental work shows that the quasisteady equation of motion (model a) describes the sphere's behaviour in an oscillating flow field with vortex separation for small values of the modified Strouhal number only. The threshold value seems to lie near 0.02 for freely movable spheres. However, for frequencies below the threshold value the quasisteady equation shows the smallest error of the three models.

For higher values of the modified Strouhal number, interference occurs in the system of the wake and the sphere, producing the wake which causes a stronger retardation of the average velocity. In this frequency regime the sphere shows regular lateral fluctuations with the frequency of the flow field, indicating a regular vortex shedding from the sphere. Near the natural vortex shedding frequency the time averaged fall velocity is observed to be up to 30% less than the value predicted by a quasisteady equation. Because of the intense interaction between the particle and the surrounding fluid, an increase of heat and mass transfer, for example in a pulsed extraction column, must be expected in this regime. Sufficiently high values of the frequency prevent the vortex shedding in the regime investigated here, which covers mainly the regular vortex sheet. The time averaged velocity is higher than expected with a quasisteady equation of motion (model a), but is less than the settling velocity in a fluid at rest. The boundary between the resonance regime and the regime of suppressed vortex shedding cannot be a function of the Reynolds number and the Strouhal number only, since the density ratio of the sphere's material and the fluid influences the trajectory in the case of lateral fluctuations.

If there is no vortex shedding from a sphere, whether the vortex shedding is suppressed or the Reynolds number based on the sphere's relative velocity is too low and therefore no vortex sheet exists, then the predicted values according to an equation of motion with a drag given by Odar (model c) transformed to the relative motion, fits the experimental data best for the three models. If the particle motion is rectilinear, the results obtained with a rigidly supported particle can be applied to the motion of a free particle. Thus, this empirical function given by Odar (1964) for the unsteady drag is found from experimental results obtained with an oscillating sphere kept between springs in a fluid at rest. The acceleration of a sphere settling in a fluid at rest (experimental results Moorman 1955; Odar 1966a,b) can be described with sufficient exactness by this equation. Furthermore, the experimental data of a freely movable sphere in an oscillating flow field deviate least from the statements of this equation of motion. Therefore it can be presumed that the application of this equation of motion holds true also beyond the mentioned cases of deterministic motions.

CONCLUSIONS

(1) In a vertical sinusoidally oscillating flow field the average velocity of a sphere is smaller than the velocity in a fluid at rest. For this type of fluid motion the sphere cannot ascend against gravity. Thus the retarding effect takes values $0 < u_0/w_f \leq 1$.

(2) A quasisteady equation of motion (model a) frequently used in engineering practice predicts the sphere behaviour in an oscillating flow field only for small values of the frequency relative to the natural vortex shedding frequency. Essentially undistorted vortex shedding, the precondition for a quasisteady state of the flow with vortex shedding from the sphere, seems to extend to $fd/w_f \lessapprox 0.02$.

(3) A frequency of the flow field sufficiently larger than the natural vortex shedding frequency causes suppression of the vortex shedding. In all cases of motion without vortex sheet in the wake, the equation of motion containing the substituted unsteady drag given by Odar (model c) predicts the sphere's motion most exactly among the three models.

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Résumé—On étudie théoriquement et expérimentalement le mouvement de particules isolées dans un écoulement fluctuant sinusoïdalement, en tant que cas le plus simple du mouvement turbulent d'un tourbillon. La connaissance complète de l'interaction entre les particules et le fluide environnant faisant défaut, on cite les équations de trois modèles d'interprétation du comportement de la sphère. On décrit les différents aspects du comportement de la particule dans un écoulement oscillant et on donne les limites de validité de l'équation du modèle qui présente le moins d'insuffisances par rapport aux résultats expérimentaux.

Auszug—Theoretische und experimentelle Ergebnisse über die Bewegung einzelner Partikeln in einem sinusförmig oszillierenden Strömungsfeld, das als eine entartete turbulente Bewegung eines Turbulenzballens aufgefaßt werden kann, werden mitgeteilt. Da die Wechselwirkungskraft zwischen einem Teilchen und der umgebenden Flüssigkeit weitgehend unbekannt ist für Reynoldszahlen größer als eins, werden drei Modellgleichungen zur Deutung des Bewegungsverhaltens herangezogen. Die verschiedenen Erscheinungsformen des Bewegungsverhaltens eines Teilchens im oszillierenden Strömungsfeld werden geschildert und die Gültigkeitsgrenzen der Bewegungsgleichung mit den geringsten Abweichungen zu den Versuchsergebnissen aufgezeigt.

Аибгиф—Исследовано теоретически и эмпирически движение обособленных частиц в поле синусоидально вибрирующей жидкости, представляющее простейший случай турбулентного движения вихря. Поскольку полнота сведений о взаимодействии таких частиц и окружающей жидкости отсутствует, в работе представлены уравнения трех моделей, интерпретирующих поведение такой сферы. В работе описаны разнообразные явления, сопровождающие поведение указанной частицы в поле вибрирующей жидкости, но ценность уравнений таких моделей, дающих наименьшие отклонения от результотов опыта, ограничена.